

## 3D and 4D data processing with quaternion tensor decompositions

**Context** The efficient processing of 3D and 4D data is pivotal in many applications such as robotics, color and polarization imaging or attitude control, among others. Such multichannel data is often represented using quaternions – a generalization of complex numbers in four dimensions - in order to simplify expressions and leverage unique geometric and physical insights offered by this algebraic representation. When 3D/4D data is measured along multiple diversities (e.g. time, frequency, space, etc.), datasets can be viewed as multidimensional quaternion arrays – also called *quaternion tensors*.

**Summary** While quaternion tensors can encode in a compact and meaningful way 3D/4D datasets, they define a challenging mathematical object for which little results are currently available. This can be explained by the noncommutativity of quaternion multiplication, which prevents a direct use of the tensor methods originally developed for real and complex tensors [1]. Yet, motivated by recent results in constrained quaternion matrix factorizations [2], we have proposed a rigorous framework to perform quaternion tensor decompositions. This M2R internship will take advantage of this new framework and develop *efficient algorithms to perform quaternion tensor decompositions*. As a first task, the candidate will focus on the quaternion canonical polyadic decomposition (CPD), a fundamental tool that allows a decomposition of quaternion tensor in rank-one terms. He will develop and compare the performances of two categories of algorithms performing quaternion CPD: (i) based on a real-constrained tensor reformulation, inspired by [3], and (ii) full-quaternion domain algorithms. In addition, he/she will apply the proposed methodology to the study of real datasets from several applications, such as color imaging (3D data) and polarization imaging (4D) data. One key objective will be to benchmark performances of quaternion tensor decompositions against standard real-domain tensor decompositions.

This internship can be followed by a Ph.D research project  
starting October, 2022, at CRAN, Nancy (more details)

**Candidate profile** He/she should be enrolled in a M2R or engineer diploma in one or more of the following fields: signal and image processing, machine learning, applied mathematics. The candidate should have good writing and oral communication skills.

**Supervision and environment** This M2R internship will be jointly supervised by Julien Flamant (CNRS research scientist), Sebastian Miron (Associate Professor at Université de Lorraine), Xavier Luciani (Associate Professor at Seatech, Université de Toulon) and Yassine Zniyed (Associate Professor at Seatech, Université de Toulon) Depending on its preferences, the candidate will be either located at CRAN, Nancy or either at LIS, Toulon.

**Please contact** Julien Flamant, julien.flamant@univ-lorraine.fr or Xavier Luciani, luciani@univ-tln.fr

## References

- [1] T. G. Kolda and B. W. Bader, “Tensor decompositions and applications,” *SIAM review*, vol. 51, no. 3, pp. 455–500, 2009.
- [2] J. Flamant, S. Miron, and D. Brie, “Quaternion non-negative matrix factorization: Definition, uniqueness, and algorithm,” *IEEE Transactions on Signal Processing*, vol. 68, pp. 1870–1883, 2020.
- [3] A. L. De Almeida, X. Luciani, A. Stegeman, and P. Comon, “Confac decomposition approach to blind identification of underdetermined mixtures based on generating function derivatives,” *IEEE Transactions on Signal Processing*, vol. 60, no. 11, pp. 5698–5713, 2012.